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THE NEAR-HORIZON GEOMETRY OF DILATON-AXION BLACK HOLES

G. CLÉMENT
LAPTH (CNRS), B.P.110, F-74941 Annecy–le–Vieux cedex, France
E-mail: gclement@lapp.in2p3.fr

D. GAL'TSOV[a]
Department of Theoretical Physics, MSU, 119899, Moscow, Russia
E-mail: galtsov@grg.phys.msu.su

Static black holes of dilaton-axion gravity become singular in the extreme limit, which prevents a direct determination of their near-horizon geometry. This is addressed by first taking the near-horizon limit of extreme rotating NUT-less black holes, and then going to the static limit. The resulting four-dimensional geometry may be lifted to a Bertotti-Robinson-like solution of six-dimensional vacuum gravity, which also gives the near-horizon geometry of extreme Kaluza-Klein black holes in five dimensions.

The discovery of the AdS/CFT dualities[1] stimulated the search for geometries containing AdS sectors, which typically arise as the near-horizon limit of BPS black holes or p-branes in various dimensions. Here we discuss the near-horizon limit of 4-dimensional black holes arising in the truncated effective theory of the heterotic string: dilaton-axion gravity with one Abelian vector field (EMDA).

The Einstein-frame metrics of the NUT-less rotating extreme black hole solutions of EMDA[2] are given by

$$ds^2 = \frac{\Sigma r^2}{\Gamma}\,dt^2 - \frac{\Gamma}{\Sigma}\sin^2\theta\left(d\varphi - \frac{2Ma(r+a)}{\Gamma}dt\right)^2 - \Sigma\left(\frac{dr^2}{r^2} + d\theta^2\right), \qquad (1)$$
$$\Sigma = h - a^2\sin^2\theta, \quad \Gamma = h^2 - r^2a^2\sin^2\theta, \quad h = r^2 + 2M(r+a),$$

M and a being the mass and the rotation parameter. The horizon $r = 0$ reducing to a point in the static case $a = 0$, we carry out the near-horizon limit in the rotating case $a \neq 0$. First, we transform to a frame co-rotating with the horizon, and rescale time by $t \to (r_0^2/\lambda)t$ ($r_0^2 \equiv 2aM$). Then, we put $r \equiv \lambda x$, $\cos\theta \equiv y$, and take the limit $\lambda \to 0$, arriving at

$$ds^2 = r_0^2\left[(\alpha + \beta y^2)\left(x^2\,dt^2 - \frac{dx^2}{x^2} - \frac{dy^2}{1-y^2}\right) - \frac{1-y^2}{\alpha+\beta y^2}(d\varphi + x\,dt)^2\right], \qquad (2)$$

with $\beta = a/2M, \alpha = 1 - \beta$. This is similar in form to the extreme Kerr-Newman near-horizon metric[3], both having the symmetry group $SL(2,R) \times U(1)$, and coinciding in the extreme Kerr case $M = a$. Now take the static limit $a \to 0$ in (2), keeping $r_0^2 = 2aM$ fixed. This yields (for $r_0^2 = 1$) the metric

$$ds^2 = x^2\,dt^2 - \frac{dx^2}{x^2} - \frac{dy^2}{1-y^2} - (1-y^2)(d\varphi + x\,dt)^2\,. \qquad (3)$$

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This metric, together with the associated near-horizon dilaton ϕ, axion κ and gauge potential in the static limit (independent of the original parameters of the black hole solution)

$$\phi = 0, \qquad \kappa = -y, \qquad A = -y(d\varphi + xdt)/\sqrt{2}, \tag{4}$$

constitute a new Bertotti-Robinson-like solution of EMDA, the BREMDA solution.

This solution has properties remarkably similar to those of $AdS_2 \times S^2$: all timelike geodesics are confined, and the Klein-Gordon equation is separable in terms of the usual spherical harmonics. The isometry group $SL(2,R) \times U(1)$, with $U(1)$ being the remnant of the $SO(3)$ symmetry of S^2, is generated by the Killing vectors L_{-1}, L_0, L_1 and $L_\phi = \partial_\phi$. The first three of these constitute the $sl(2,R)$ subalgebra of an infinite-dimensional algebra of asymptotic symmetries of (3), with generators

$$L_n = t^{-n}\left[\left(t + \frac{n(n-1)}{2x^2 t}\right)\partial_t \, + \, x(n-1)\partial_x - \frac{n(n-1)}{xt}\partial_\phi\right] \tag{5}$$

(where $n \in Z$) satisfying the Witt algebra $[L_n, L_m] = (n-m)L_{n+m}$ up to terms $O(x^{-4})$. It can be expected that a representation in terms of asymptotic metric variations will lead to the Virasoro extension of this algebra with a classical central charge, opening the way for microscopic counting of the horizon microstates of dilaton-axion black holes.

The BREMDA solution does have a close connection with $AdS_2 \times S^2$, which is discovered by lifting it to 6 dimensions. EMDA in 4 dimensions may be shown [4] to derive from 6-dimensional vacuum gravity with 2 commuting spacelike Killing vectors ∂_η, ∂_χ via a two-step Kaluza-Klein reduction together with the assumption of a special relation between the Kaluza-Klein gauge fields, according to the ansatz

$$\begin{gathered} ds_6^2 = ds_4^2 - e^{-\phi}\,\theta^2 - e^{\phi}\,(\zeta + \kappa\theta)^2, \\ \theta = d\chi + A_\mu dx^\mu, \quad \zeta = d\eta + B_\mu\, dx^\mu, \quad G_{\mu\nu} = e^{-\phi}\tilde{F}_{\mu\nu} - \kappa F_{\mu\nu} \end{gathered} \tag{6}$$

($F = dA$, $G = dB$). Using this ansatz, the BREMDA solution (3)-(4) may be lifted and rearranged, yielding the vacuum solution BR6 of 6-dimensional gravity

$$ds_6^2 = x^2 dt^2 - \frac{dx^2}{x^2} - \frac{dy^2}{1-y^2} - (1-y^2)d\varphi^2 - (d\chi_+ - \sqrt{2}xdt + \sqrt{2}yd\varphi)^2 - d\chi_-^2. \tag{7}$$

This is the trivial 6-dimensional embedding of a 5-dimensional metric BR5, which can be shown to be the common near-horizon limit of all static, NUT-less black holes of 5-dimensional Kaluza-Klein theory. It enjoys the higher symmetry group $SL(2,R) \times SO(3) \times U(1) \times U(1)$, but breaks all fermionic symmetries: no covariantly constant spinors exist.

References

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